Addenda and Errata to "An Introduction to Stochastic Thermodynamics"

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1 Major changes

Errata on the convergence theorem (continuous time) in Sec.2.3

The present version (ver.1) of the textbook, the proof of the convergence theorem for continuous time (Sec.2.3, p27) has a logical gap [1]. Here we present an alternative proof for the theorem.

Claim. *Let R be a $K \times K$ transition rate matrix with connectivity. Then, there exists $\kappa > 0$ such that for any finite t less than κ ($\kappa > t > 0$), all the matrix elements of e^{tR} is strictly positive.*

Since $e^{tR}e^{t'R} = e^{(t+t')R}$ holds, it suffices to prove this theorem for our purpose.

Proof. Owing to the connectivity of R, for any elements (i,j) there exists $m \geq 0$ such that $(R^n)_{ij} = 0$ for $n \leq m-1$ and $(R^m)_{ij} > 0$. Here, $(R^m)_{ij}$ is positive, because this term consists of the sum of products of strictly positive off-diagonal elements of R (the shortest paths from j to i). Then, the (i,j) elements of e^{tR} can be written as

$$(e^{tR})_{ij} = \sum_{n=0}^{\infty} \frac{t^n}{n!}(R^n)_{ij} = \sum_{n=m}^{\infty} \frac{t^n}{n!}(R^n)_{ij} = \frac{t^m}{m!}\left((R^m)_{ij} + f(t)\right), \tag{1}$$

where $f(t)$ is a continuous function with $f(0) = 0$. The existence (i.e., convergence of infinite series) and continuity of $f(t)$ follows from the convergence and continuity of e^{tR}.

Since $f(t)$ is continuous and $f(0) = 0$, there exists κ_{ij} such that $\kappa_{ij} > t > 0$ implies $(R^m)_{ij} + f(t) > 0$. By setting $\kappa = \min_{i,j} \kappa_{ij}$, we arrive at the desired result. □

Acknowledgement
I thank Hal Tasaki for pointing out the logical gap of the proof in the present version of the textbook.

2 Minor corrections

Chap.17

- The right-hand of (17.51): $\infty \to 1$.

Chap.18

- The right-hand side of (18.83):

$$\langle \delta E \rangle_\tau := \frac{1}{\tau}\int_0^\tau dt \left(\mathrm{Tr}[H(t)^2\rho(t)] - \mathrm{Tr}[H(t)\rho(t)]^2\right) \qquad \text{(incorrect)}$$

$$\langle \delta E \rangle_\tau := \frac{1}{\tau}\int_0^\tau dt \sqrt{\mathrm{Tr}[H(t)^2\rho(t)] - \mathrm{Tr}[H(t)\rho(t)]^2} \qquad \text{(correct)}$$

- The explanation above (18.86) is incorrect (The correct direction of the inequality is its opposite:). We replace the description around (18.86) as follows:

  Noting that $\arccos x$ behaves as $\sqrt{2(1-x)}$ around $x \simeq 1$, we have a key relation

$$\begin{aligned}\mathcal{L}(|\psi(t)\rangle, |\psi(t+\Delta t)\rangle) &= \sqrt{2(1 - |\langle\psi(t)|\psi(t+\Delta t)\rangle|)} + o(\Delta t) \\ &= \frac{\sqrt{\Delta E^2}}{\hbar}\Delta t + o(\Delta t).\end{aligned}$$

- The right-hand side of (18.89) and (18.92): $\frac{\langle \delta E\rangle_\tau}{\hbar} \to \frac{\langle \delta E\rangle_\tau}{\hbar}\tau$.

Acknowledgement
I thank Mr. Kunimi for pointing out several improper descriptions.

[1] In the textbook, $(I + \Delta tR)^N$ is considered and shown that all the matrix elements are strictly positive for large N and finite Δt. However, the above argument does not cover the case of $\Delta t \to 0$ with $N \to \infty$.